

Human Sexual Contact Network as a Bipartite Graph

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A simple model to encapsulate the essential growth properties of *the web of human sexual contacts* is presented. In the model only heterosexual connection is considered and represented by a random growing bipartite graph where both male-female contact networks grow simultaneously. The time evolution of the model is analysed by a rate equation approach leading to confirm that male and female sexual contact distributions decay as power laws with exponents depending on influx and charisma of the sexes.

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I. INTRODUCTION

Yet another human interaction network! *The web of human sexual contacts* is introduced in [1], where the authors make extensive analysis of 1996 Swedish survey of sexual behaviour and show that the cumulative distributions of number of sexual partners for males and females have power-law forms with exponents $\alpha_f \approx 2.54$ for females and $\alpha_m \approx 2.31$ for males, during a single year and $\alpha_{f_{tot}} \approx 2.1$, $\alpha_{m_{tot}} \approx 1.6$ for entire life time.

Human sexual behaviour is a subject area [2] in itself and beyond the scope of this work. Here we will concentrate on capturing the essential details needed to form such a social network for which we will make use of the developments from the previous network models [3–9] and general consensus about human sexual behaviour.

The model introduced in [3] has identified two basic principles that seem to govern the growth dynamics of human interaction networks, namely preferential attachment and continuous growth. This differs from the traditional complex network consideration [10] of connecting together a fixed number of network elements (nodes) which results in Poisson distributions of the connectivity.

The usual approach in most complex network models [8,9,11,12] that have human attributes is to employ the preferential attachment process; an incoming node is more likely to connect to a node with higher connections. This process yields power-law distributed connectivity in the network. A similar process takes place in a sociophysics model [13], where probability of winning a fight depends on the number of victories, consequence of which is a self-organised hierarchy in the system. Again a well studied biological evolution model [14], in which the power-law behaviour is due to increasing fitness of the species.

Contrary to previous human interaction network models, the model we consider here has two different types of nodes and the linking process is discriminative as well as preferential. In the proceeding section, the model is built by connecting new nodes to nodes with higher connections and analysed by a rate equation approach. The results and the discussions are presented in the final section.

II. THE MODEL

The growth process of the sexual contact network described by a random growing bipartite graph (illustrated in Fig. 1) considers the following:

- (i) With probability q a new (no previous connection) female is connected to an existing male, which contributes to the number of females with contact degree $l = 1$ in the network and increases the degree k of the connecting male.
- (ii) With probability p a new male is connected to an existing female, which has the analogous effect as in (i).
- (iii) With probability r a link is created between an existing male and an existing female, which increases the degrees of both sexes.

In addition to above rules, initially there must be at least one connected male-female pair and only heterosexual contact is allowed among the sexes. Also the process r only counts once between the same pair; that is once a link between two individuals is created it remains and further contact between them does not increase their contact degrees.

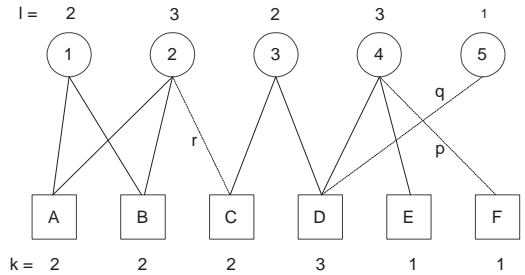


FIG. 1. Schematic illustration of the growth processes of male-female sexual contact network. Male nodes (squares) labelled A to F and female nodes (circles) labelled 1 to 5. The numbers above the nodes are indicating the node degree l for females and k for males, i.e. female node 1 has degree 2 and male node D has degree 3.

We define male sexual degree distribution A_k as the number of males with k female connections. The rate at which a male attracts females is proportional to his number of connections and it grows linearly with increasing k ; in other words the connection probability of a male increases as he becomes more experienced. And similarly does the female sexual degree distribution B_l with attraction rate proportional to l . However, it seems necessary to consider some characteristics unique to both sexes so that we may underpin the social expectations about their sexual contacts. Here we introduce λ and μ as the *charisma* parameters of males and females respectively. These parameters should not be confused with *charm* or *beauty*, which could be used as attractiveness of individuals. Here we are considering only the general attractiveness of the sexes, which are shaped by social expectations.

The charisma parameters are added into the rate of connections given $k + \lambda$ for males and $l + \mu$ for females. Since we need $k + \lambda > 0$ and $l + \mu > 0$ for the network to grow, λ and μ are considered to be positive integers and small compared to the contact degrees of k and l . Also, the general consensus is that males tend to exaggerate their number of sexual partners. This desire puts females in demand and increases female charisma. Conversely, the social expectation of females restricts them to report less and partly because of this they are more selective about their sexual partners. Using this information we argue that the rate at which females attract sexual partners should be greater than that for males and therefore $\mu > \lambda$.

In this model we are considering a large system where the random fluctuations in the linking process can be neglected when compared with the collective behaviour of the individuals. Therefore it is appropriate to use mean-field approximation. Then we can write the time evolution of the average number of males with k contacts $A_k(t)$ as

$$\frac{dA_k(t)}{dt} = \frac{q+r}{M_a} [(k-1+\lambda)A_{k-1}(t) - (k+\lambda)A_k(t)] + p\delta_{k1} \quad (1)$$

and similarly for females with l contacts,

$$\frac{dB_l(t)}{dt} = \frac{p+r}{M_b} [(l-1+\mu)B_{l-1}(t) - (l+\mu)B_l(t)] + q\delta_{l1}. \quad (2)$$

Since the structure of Eq. (1) and Eq. (2) are the same we will only analyse the former equation and apply results to the latter.

The first term in the square brackets indicates the contribution to A_k when a male of $k-1$ connections acquires a new contact, the corresponding losses are given by the second term. The growth of the network due to influx is expressed by the last term, with p being a probability of

addition of males with $k = 1$ connection. The multiplicative factor M_a is to ensure the appropriate normalization and is given by

$$M_a(t) = \sum_k (k + \lambda) A_k(t). \quad (3)$$

The total number of males in the system is $A(t) = \sum_k A_k(t)$ and the total number of females $B(t) = \sum_l B_l(t)$. From initial conditions we have $A(t) = 1 + pt$ and similarly $B(t) = 1 + qt$ making an assumption that there is one of each sex representatives before the growth commences. In particular, the bipartite structure [15,16] of the graph ensures that the total number of connections in the system is the same for both males and females, which means that $L(t) \equiv K(t) = \sum_k k A_k(t) = t + 1$, with one link before the first time step. Also, the network does not contain monogamous individuals after the first time step.

We start our analysis by looking at the moments of $A_k(t)$, which are defined as

$$M_i(t) \equiv \sum_k k^i A_k(t). \quad (4)$$

For the first few moments it is easy to show that

$$M_0(t) = M_0(0) + (q + r + p)t \quad (5)$$

and

$$M_1(t) = M_1(0) + pt. \quad (6)$$

For large times the initial values of the moments become irrelevant so that we get

$$M_a(t) = [q + r + p(1 + \lambda)]t, \quad (7)$$

which is a linear function of time and indicates that Eq. (1) is also linear in time. Using this input we can write $M_a(t) = m_a t$ implying $m_a = q + r + p(1 + \lambda)$ and $A_k(t) = a_k t$. Substituting these relations in Eq. (1) leads to a time independent recurrence relation

$$\left(\frac{m_a}{q+r} + k + \lambda \right) a_k = (k-1+\lambda)a_{k-1} + \frac{m_a}{q+r} p \delta_{k1}. \quad (8)$$

Iterating Eq. (8) we obtain

$$a_k = \frac{\Gamma(k + \lambda)}{\Gamma(\frac{m_a}{q+r} + k + \lambda + 1)} \frac{\Gamma(\frac{m_a}{q+r} + \lambda + 1)}{\Gamma(\lambda)} \frac{m_a}{q+r} p. \quad (9)$$

Using the former relation for the case of females, by simply replacing the relevant parameters gives

$$b_l = \frac{\Gamma(l + \mu)}{\Gamma(\frac{m_b}{p+r} + l + \mu + 1)} \frac{\Gamma(\frac{m_b}{p+r} + \mu + 1)}{\Gamma(\mu)} \frac{m_b}{p+r} q. \quad (10)$$

For large k and l Eq. (9) and Eq. (10) are asymptotically equivalent to

$$a_k \sim k^{-(1+\frac{m_a}{q+r})} \quad (11)$$

and

$$b_l \sim l^{-(1+\frac{m_b}{p+r})}. \quad (12)$$

The distributions $a_k \sim k^{-\gamma_m}$ and $b_l \sim l^{-\gamma_f}$ scale as power-laws, which is a vulnerable structure for social networks as far as the epidemics and the spreading of computer viruses are concerned [17–19]. The scaling exponent

$$\gamma_m = 2 + \frac{p(1+\lambda)}{1-p} \quad (13)$$

depends upon male charisma parameter λ and the rate p of addition of males into the network. Similarly, the exponent

$$\gamma_f = 2 + \frac{q(1+\mu)}{1-q} \quad (14)$$

depends upon female charisma parameter μ as well as the rate of arrival of females. However, since we have the constraint $p + q + r = 1$ implying one link at each time step, the growth of one network is effected by the growth of the other.

III. DISCUSSION AND CONCLUSIONS

From the definition of the model, we have four free parameters, $0 \leq p \leq 1$, $0 \leq q \leq 1$, $\mu > \lambda > 0$ and the fifth one is given by the relation $(p + q + r) = 1$.

Starting with a simple example, let us ignore the process r and take $p + q = 1$. This yields

$$\gamma_f = 2 + \frac{q(1+\mu)}{1-q} \quad \text{and} \quad \gamma_m = 2 + \frac{(1-q)(1+\lambda)}{q}. \quad (15)$$

We get $\gamma_f \rightarrow 2$, $\gamma_m \rightarrow \infty$ as $q \rightarrow 0$ and $\gamma_f \rightarrow \infty$, $\gamma_m \rightarrow 2$ as $q \rightarrow 1$. Although this is an unrealistic assumption, it simply illustrates the strong coupling of the influxes of males and females into the system. A similar strong coupling is also observed in [20] where a more general coupled growing network is considered.

If we pick $\lambda = 2$, $\mu = 3$ and substitute into the scaling exponents in Eq. (13) and Eq. (14) respectively we get $p \approx 0.30$ and $q \approx 0.28$ for the corresponding power-law exponents in [1]. Where $\gamma_m = 1 + \alpha_m = 3.31$ and $\gamma_f = 1 + \alpha_f = 3.54$ for the short time behaviour analysis. In this example choice of values for λ and μ allowed us to obtain $p \approx q$ meaning that the increase of links due to new males and females is approximately the same and about 40 % of the links are created among the pre-existing males and females. The average number of partners of the sexes $\sum_k k A_k / \sum_k A_k = 1/p \approx 3$ is the same since we have a close system.

Let us now consider the distribution of contacts during the life time of the sexes. Here, we introduce another

parameter β to the attachment rates to describe bad experiences, aging or inactivity of the sexes and hence the new parameter is a negative input. It can be in the range $[0, k + \lambda)$ or $[0, l + \mu)$, which seems reasonable to assume for $k, l > 2$. We can denote $\lambda_1 = \lambda - \beta_m$ and $\mu_1 = \mu - \beta_f$ where we assign a unique β parameter to each kind to discriminate their sexual behaviour. The new scaling exponents

$$\gamma_f = 2 + \frac{q(1+\mu_1)}{1-q} \quad (16)$$

and similarly

$$\gamma_m = 2 + \frac{p(1+\lambda_1)}{1-p}. \quad (17)$$

Using the same values we used earlier for the free parameters gives $\gamma_f = 1 + \alpha_{f_{tot}} = 3.1$ for $\beta_f \approx 1.17$ the same for all females and $\gamma_m = 1 + \alpha_{m_{tot}} = 2.6$ for $\beta_m \approx 1.6$ for all males. Consequently we have $\beta_f < \beta_m$ implying that the sexual connections of males tend to suffer more from aging, bad experiences or inactivity than that of females. Perhaps this may be the reason why males resort to lies to maintain self image.

In conclusion, our simple model seems adequate to simulate the dynamics of the web of human sexual contacts. The gratifying feature of rate equation approach is that it gives flexibility to include diverse range of parameters to model any situation within the frame of this work. The charisma parameters used here are shaped by the western social expectations and they can be changed to reflect the views of other societies. Also, one can introduce *fitness*, multiplicative quenched disorder into the system as in [12] to account the activity level of different age groups. Finally, a challenging issue to address would be to study male-female degree correlations of this model.

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